

Convert the system  $\begin{cases} y_1' = y_1 + y_2 & (1) \\ y_2' = 2y_1 + 3y_2 & (2) \end{cases}$

into a single ODE:

From (2), we get

$$y_1 = \frac{y_2' - 3y_2}{2} \quad (*)$$

Substitute this  $y_1$  into (1):

$$\underbrace{\frac{y_2'' - 3y_2'}{2}}_{y_1'} = \underbrace{\frac{y_2' - 3y_2}{2}}_{y_1} + y_2$$

Simplify:  $y_2'' - 4y_2' + y_2 = 0$

Once you solve for  $y_2$  from this ODE, you substitute  $y_2$  into (\*) to get  $y_1$ .